Cognitive Biases are Critical in Conflict Bargaining

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Abstract

This paper explores how cognitive biases affect conflict bargaining by integrating prospect theory into classical bargaining models. While traditional bargaining theory predicts that conflict is inefficient and should be avoidable when there are positive costs to fighting, this paper demonstrates that when actors exhibit cognitive biases consistent with prospect theory (loss aversion, diminishing sensitivity, and probability weighting), conflict can become unavoidable even without information asymmetries or commitment problems. The model shows that conflicting reference points, combined with these cognitive biases, can make conflict inevitable. When players' reference points align with their preferred outcomes, even modest cognitive biases can eliminate any possibility of peaceful settlement. This suggests that focusing solely on reducing information asymmetries or improving commitment capabilities may be insufficient for preventing conflict if the underlying cause is rooted in cognitive biases and conflicting reference points. The findings indicate that successful conflict resolution may require strategies aimed at shifting the reference points of the involved parties.

1 Introduction

Conflict is nearly ubiquitous in human society. Many models have been made in the social sciences to explain the existence of conflicts and thereby the methods through which they could be eliminated. In this paper I develop a model that demonstrates how conflict can be a result of cognitive biases of the clashing parties. In particular, I use a game theoretic model popular in the international relations literature known as bargaining theory and I replace the rational actors with actors that make decisions according to prospect theory. I then analyze the interacting effects of the different cognitive biases inherent in prospect theory on the likelihood of conflict in the model.

The main result of bargaining theory predicts that conflict is inefficient. Given a positive fixed cost to conflict, conflict is a costly gamble and there always exists a range of outcomes that are preferable to both sides. Conflict, therefore, must be caused by some distortion, such as private information or inability to commit to settlements. How does this prediction change if players act not as perfectly rational actors, but instead embody cognitive biases? As I will show, players who act according to the predictions of cumulative prospect theory can make decisions radically different than those that have standard risk-neutral or risk-averse preferences. With this model, conflict can occur as a product purely of the player's preferences (and/or probability weighting), without any need for distortions.

This finding comes with an important implication: attempts to eliminate these distortions may not resolve the conflict if the cause of the conflict is rooted in cognitive biases. It also suggests new methods through which conflict can be avoided. In the analysis below, we will find that a key factor in the possibility of conflict is the reference point of each player, which defines which outcomes the player views as "gains" and which as "losses". As each player's reference point approaches her preferred outcome, conflict becomes more likely, and within a reasonable range of parameters, can become unavoidable. Thus, interventions aimed at altering the reference points of the players may be an effective method to avoid or end otherwise intractable conflicts.

In section 2, I will present bargaining theory and a simple modification that embodies the core results of this paper. I will use this model to show how each of the parameters of prospect theory affect the possibility of conflict. Additionally, I will present a proposition stating that when the players' reference points are diametrically opposed, there always exists some positive level of cost to conflict at which conflict is unavoidable. This contrasts with the original model of bargaining theory in which conflict never occurs if conflict entails any positive fixed cost.

Then, in section 3, I will present the full model, completely integrating prospect theory into conflict bargaining. Again, the effects of the parameters on the possibility of conflict are closely examined. Though the effects are now more nuanced, the original intuition of the importance of cognitive biases remains. I will also prove a second proposition demonstrating the convergence of each player's indifference point (between conflict and negotiated settlement) into their reference point. This convergence occurs via the interacting effect of the three important parameters in prospect theory, namely the level of loss aversion, the level of risk seeking in the loss domain (diminishing sensitivity), and the strength of probability weighting. Thus, I will show the key importance of the reference points, how this is dependent upon cognitive biases found in prospect theory and how conflict is more likely to occur when each player's reference point is closer to their own preferred outcome than the other player's reference point.

Finally, in section 4, I will discuss the implications of this model for predicting and avoiding conflicts.

2 Simple Model

In this section, I will present a simple adaptation of bargaining theory as presented in Fearon (1995) using prospect theory. This model will use strong assumptions to simplify the presentation, with the full model being deferred to the next section. In particular, each of the players' reference points is assumed to be their prefered outcome. First, however, I will begin by reviewing Fearon's original model.

2.1 Fearon's Classic Model of Bargaining

Fearon (1995) models a situation of two players with diametrically opposed preferences over a continuum of outcomes that are normalized to the unit interval. Though he identifies the players as sovereign states, there is nothing in the model that cannot be generalized to conflict more broadly between any two strategic actors. The two sides can either come to a mutually agreed upon outcome through negotiation or can attempt to force their preferred outcome through conflict, which has a fixed cost for each side.

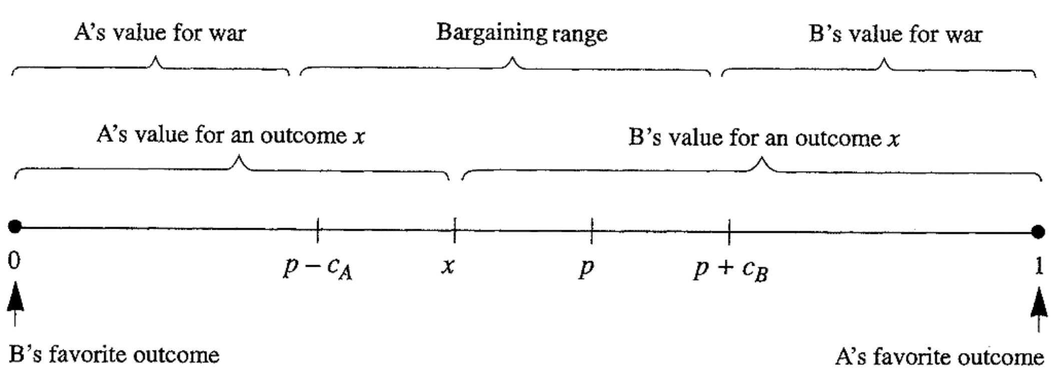
Formally, the game is defined by the following elements:

- Two players: $I = \{A, B\}$.

- VNM utility functions for players A and B. respectively: $u_A(x)$ and $u_B(1-x)$ that are both continuous, increasing and weakly concave such that $u_i(1) = 1$ and $u_i(0) = 0$ for $i \in \{1, 2\}$.
- A probability p that player A will win in the event of conflict.
- A fixed cost that each side pays in the event of conflict, c_A and c_B respectively.

Fearon does not explicitly model the players' actions in this basic model, but we can generalize them as follows: A and B simultaneously select an outcome $x_i \in [0,1], i \in \{A, B\}$. If $x_A = x_B$ then that outcome is selected, otherwise conflict occurs and the outcome 1 is selected with probability p and the outcome 0 is selected with the probability $1-p$. Figure 1, which is reproduced from Fearon (1995)'s Figure 1, illustrates the range of outcomes.

Figure 1: Outcomes from Fearon's Classic Bargaining Model



An important finding of Fearon (1995) is that as long as $c_A, c_B > 0$ and given the assumptions above, there always exists a subset of outcomes that both sides prefer to conflict. In the case of risk-neutral players, this corresponds to the range $[p - c_A, p + c_B]$ such as in Figure 1. We refer to this range as the *bargaining range* (BR) and to every outcome within the bargaining range there exist a pair of strategies that lead to a Nash equilibrium with such an outcome.[1] The guaranteed existence of a bargaining range necessitates the question: why do we often see conflict in the world? Fearon explains this by introducing distortions into his model such as private information and inability to credibly commit to future action. Though these certainly are important factors of conflict, in the next subsection, I will show that the bargaining range can disappear even without these factors if players make decisions as determined by prospect theory as opposed to rational decision theory, thus making conflict unavoidable.

2.2 A Simple Model: Preferred Outcomes as Reference Points

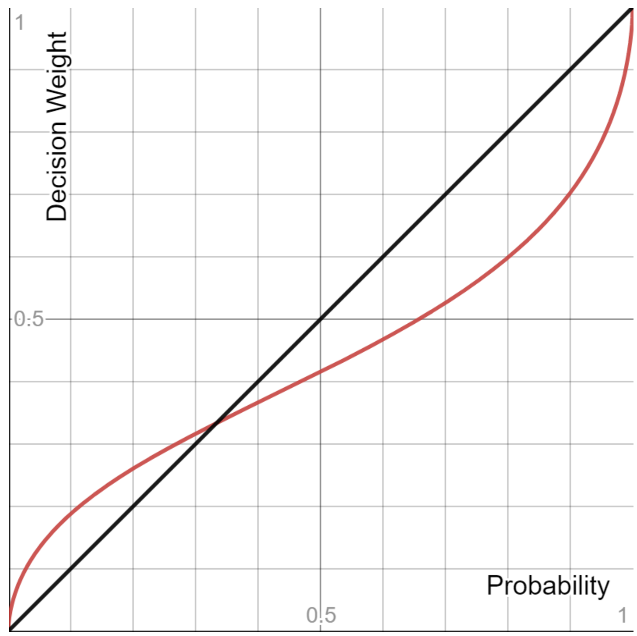
In this subsection I will introduce a game that is identical in structure to Fearon's bargaining game above. The only difference is that the players' utility functions are replaced with value functions from cumulative prospect theory, and when calculating the utility of a risky prospect, decision weights are used instead of probabilities. To motivate this discussion, I will begin with a short synopsis of cumulative prospect theory. A more complete discussion can be found in Tversky and Kahneman (1992).

The main features of cumulative prospect theory are as follows. There is a reference point (often normalized to zero) that gives a utility of zero. Outcomes above this reference point are known as the *gain domain* and outcomes below it as the *loss domain*. Agents are risk-averse within the gain domain and risk-seeking within the loss domain. In addition, agents are loss-averse, meaning that a loss of a given size lowers utility by more than a gain

[1] In this equilibrium both players select that outcome. Since it is in the bargaining range, neither player has an incentive to deviate.

of the same size raises it. Lastly, decision weights are weighted probabilities that follow an "inverse s" shape as shown in Figure 2. The model is motivated by the empirical findings that individual decision makers often embody these cognitive biases. Some studies such as Levy (1996) provide evidence that players embody these same biases in their decisions.

Figure 2: Inverse S Shaped Decision Weights



For our simple model, let A's reference point be 1 and let B's reference point be 0. Note that these are the preferred outcomes of each player. This models a situation where each side feels entitled to the entire pie. Though not a universal phenomenon, it mirrors some important scenarios. For example, this could be used to model the Israeli-Palestinian conflict, where (some elements) of each side feel entitled to the entire territory of the British Mandate for Palestine. In section 3, I will remove this assumption and explore the results of the model for a variety of reference points.

Mathematically, let:

$$v_A(x) = -\lambda(1-x)^\beta, \quad v_B(x) = -\lambda x^\beta$$

$$w(p) = \frac{p^\delta}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta}}}$$

Where $v_i(x)$ is the value of outcome x to player $i \in \{A, B\}$ and $w(p)$ is the decision weight given to probability p. β determines each player's level of risk-seeking. A lower β indicates more risk-seeking behavior. λ determines each player's level of loss aversion. A higher λ indicates a higher level of loss aversion. δ controls the probability weighting, with a lower δ leading to a more pronounced "inverse-s" shape. Note that to simplify, both players have been given the same level of loss aversion, risk-seeking and probability weighting. In addition, the value functions have been defined only for the loss domain, as no outcome is in either player's gain domain.

To solve for behavior, I will find the outcomes at which each player is indifferent between conflict and acquiescence.

$$U_A(\text{peace}) = -\lambda(1-x)^\beta = w(p)(-\lambda(1-1)^\beta) + w(1-p)(-\lambda(1-0)^\beta) - c_A = U_A(\text{conflict}) \Leftrightarrow$$
$$x_A = 1 - \left[\frac{(1-p)^\delta}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta}}} + \frac{c_A}{\lambda}\right]^{\frac{1}{\beta}} \tag{1}$$

$$U_B(\text{peace}) = -\lambda x^\beta = w(p)(-\lambda \cdot 1^\beta) + w(1-p)(-\lambda \cdot 0^\beta) - c_B = U_B(\text{conflict}) \Leftrightarrow$$
$$x_B = \left[\frac{p^\delta}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta}}} + \frac{c_B}{\lambda}\right]^{\frac{1}{\beta}} \tag{2}$$

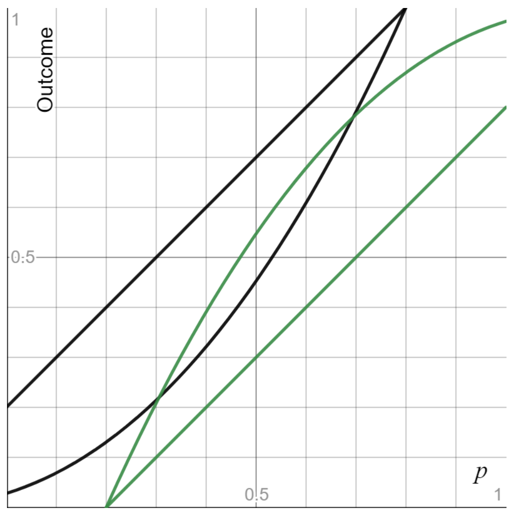
Thus, the bargaining range is equal to $[x_A, x_B]$ if $x_A \leq x_B$ and is an empty set if $x_A > x_B$. Notice that if $\beta = \lambda = \delta = 1$ then $x_A = p - c_A$ and $x_B = p + c_B$ which is identical to the risk-neutral case in subsection 2.1. However, unlike Fearon's model, the bargaining range can now be an empty set, making conflict unavoidable. What set of parameters can lead to this conclusion?

It is easy to see that λ alone cannot change the results of Fearon (1995). Keeping $\beta = \delta = 1$, (1) and (2) reduce to $x_A = p - \frac{c_A}{\lambda}$ and $x_B = p + \frac{c_B}{\lambda}$, respectively. Thus $\lambda > 1$ can reduce the bargaining range, potentially exacerbating the issues in Fearon (1995) or the issues below, but cannot explain conflict by itself.

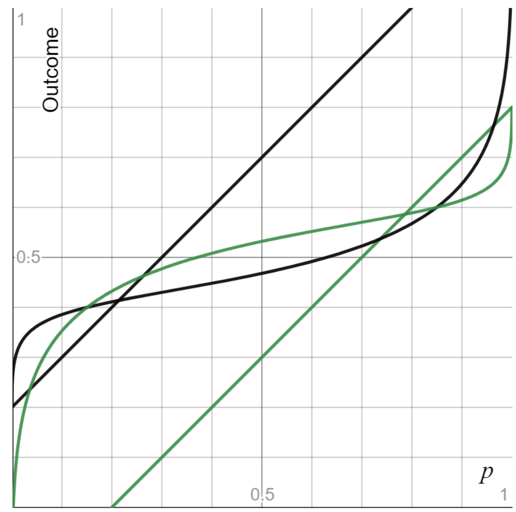
Either β or δ can explain conflict by themselves. This can be seen in Figure 3a and Figure 3b which use $\beta = 0.45$ and $\delta = 0.4$ respectively while keeping the other parameters equal to 1. We will see many figures such as Figure 3 so I will explain here in detail how to interpret them. The lines in the two subfigures of Figure 3 indicate the outcome (on the y-axis) for which player A (the green line) and player B (the black line) are indifferent between conflict and acquiescing for each value of probability p (on the x-axis). Thus, the bargaining range for each level of probability (if it is non-empty) is the vertical space between the green line and the black line. If the green line lies above the black line, that indicates that there is no outcome that both players prefer to conflict, and therefore the bargaining range for that value of p is the empty set.

The linear lines represent risk-neutral players acting according to rational decision theory. As we can see, for each level of probability, the black line lies above the green line, which means that a non-empty bargaining range always exists. The curved lines represent

Figure 3: Changing One Parameter



(a) $\beta = 0.45$



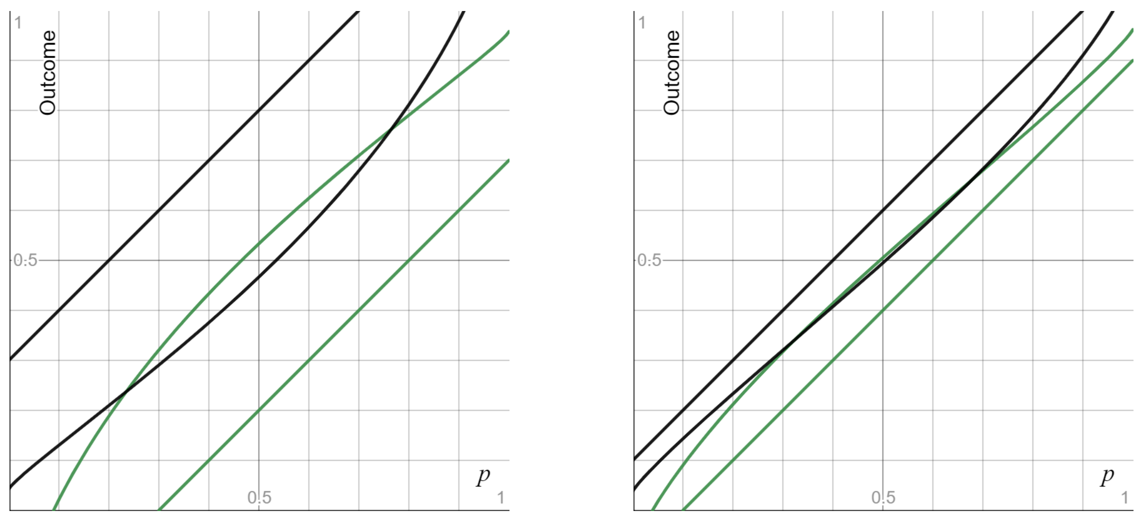
(b) $\delta = 0.4$

Note: The x-axis shows the probability of player A winning in the event of conflict and the y-axis shows the outcome. Each line gives the indifference point for a player between conflict and a negotiated resolution at that outcome level for each probability p. The linear lines refer to risk neutral agents and the curves refer to the agents under prospect theory. The green lines refer to player A and the black lines refer to player B. If the black line is above its green counterpart, then the bargaining range is the set of outcomes between the lines, and if the green line is above the black then the bargaining range is the empty set.

the preferences described in the previous paragraph. For both sets of preferences, there exists a range of probabilities such that the green line is above the black line and therefore the bargaining range is the empty set for those levels of p. We find that by manipulating just β or δ we can eliminate the bargaining range and therefore make conflict unavoidable.

It is more likely however, that conflict is caused by a combination of these parameters, as opposed to one parameter at a relatively extreme value. Figure 4 displays the effect of simultaneously changing multiple parameters. In Figure 4a, $\beta = 0.6$, $\lambda = 2$, $\delta = 0.8$, and $c_A = c_B = 0.3$. We find that for this set of parameters we have a complete breakdown of the bargaining range for a large range of probabilities p. This shows that conflict is possible even given relatively large costs. When costs are smaller, such as in Figure 4b, conflict becomes possible for an even more modest set of parameters. In Figure 4b $\beta = 0.83$, $\lambda = 1.5$, $\delta = 0.85$, and $c_A = c_B = 0.1$. Though the effect is less visually pronounced, there still remains a complete breakdown of the bargaining range between the probabilities of (approximately) $\frac{1}{3}$ to $\frac{2}{3}$.

Figure 4: Changing Multiple Parameters



(a) $\beta = 0.6,\ \lambda = 2,\ \delta = 0.8$

(b) $\beta = 0.83,\ \lambda = 1.5,\ \delta = 0.85$

Note: For explanation of figure, see Figure 3.

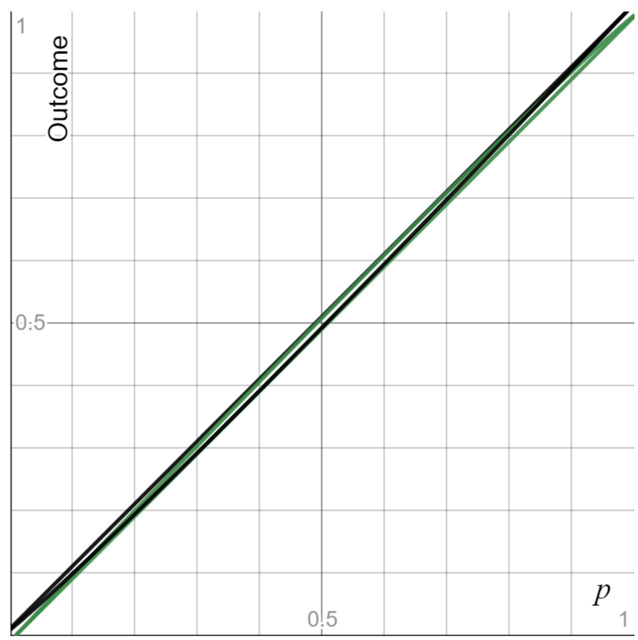
As a generalization of the last statement, we have the following proposition:

Proposition 1. *If the reference point of player A is one and the reference point of player B is zero, then for all* $0 \leq \beta, \delta \leq 1$, $\lambda \geq 1$, *and* $0 < p < 1$, *as long as either* β *or* δ *(or both) is strictly less than one, then there exist* c_A, $c_B > 0$ *such that* $BR = \emptyset$.

Proof: In appendix.

That is, for all probabilities p, as long as either β or δ is strictly less than one, even if it is very close to one and even if the other parameters are equal to one, there exist some levels of cost at which the bargaining range is the empty set and therefore conflict is unavoidable. This is in stark contradiction to Fearon's model, in which for all positive levels of cost, no matter how small, the bargaining range is non-empty. The implication of this is that with low enough costs of conflict, even very modest cognitive biases can make conflict unavoidable. An example of conflict with low costs, with λ and δ equal to one and with β only slightly less than one can be seen in Figure 5.

Figure 5: conflict With β Close to One ($\beta = 0.95$, $c_A = c_B = 0.01$)



Note: For explanation of figure, see Figure 3.

Using a simple adaptation of Fearon's classic bargaining model, I have shown that when the reference point of each player is its preferred outcome, conflict can emerge even without the introduction of other factors such as private information or commitment issues. This result is not surprising. Since both players are always in their loss domain, they are both risk-seeking, which violates the assumptions of Fearon's model. Similar results could have been reached simply by supplying risk-seeking agents, without using prospect theory.[2]

Nonetheless, the simple model is useful for multiple reasons, the first being that its simplicity makes the effects of using prospect theory clear, including changing λ or δ, whereas using standard risk-seeking agents is akin to only changing β. In addition, it can be used to model some of the most interesting and difficult conflicts, such as the Israeli-Palestinian conflict, where any concession is viewed as a loss. The model holds important real world implications. In such a conflict, efforts to make private information public or to make commitments more credible may be fruitless. Instead, preventing conflict may require changing the reference points of the actors, which brings us to the next section in which the assumptions on the reference points of the players are relaxed.

3 Full Model

I will start in subsection 3.1 by introducing the effect of changing the reference points of the players into the model and deriving the bargaining range under such a model. In subsection 3.2, I will showcase a number of interesting properties of this model.

[2]Though the disappearance of the bargaining range via probability weighting with risk neutral agents ($\beta = 1$) is a new finding.

3.1 Deriving The Bargaining Range

Let r_i be the reference point of player $i \in \{A, B\}$ and let α signify the level of risk-aversion of each player in the gain domain. The other parameters remain as defined in subsection 2.2. The value functions for each player become:

$$v_A(x|r_A) = \begin{cases} (x - r_A)^\alpha & \text{if } x \geq r_A \\ -\lambda(r_A - x)^\beta & \text{otherwise} \end{cases}, \quad v_B(x|r_B) = \begin{cases} (r_B - x)^\alpha & \text{if } x \leq r_A \\ -\lambda(x - r_B)^\beta & \text{otherwise} \end{cases}$$

$w(p)$ remains as defined in subsection 2.2. Note that in Tversky and Kahneman (1992) decision weights are defined separately for the loss and the gain domain as $w^-(p)$ and $w^+(p)$. For simplicity, I assume $w^-(p) = w^+(p) \equiv w(p)$.

The separation of the value functions into gain and loss domains makes finding the point of indifference slightly more complicated. If the utility from conflict is positive ($w(p)(1 - r_A)^\alpha - \lambda \cdot r_A^\beta \cdot w(1 - p) - c_A \geq 0$) then the indifference point must be in the gain domain. Thus we have:

$$(x_A - r_A)^\alpha = w(p)(1 - r_A)^\alpha - \lambda \cdot r_A^\beta \cdot w(1 - p) - c_A \Leftrightarrow$$
$$x_A = r_A + \left(w(p)(1 - r_A)^\alpha - \lambda \cdot r_A^\beta \cdot w(1 - p) - c_A\right)^{\frac{1}{\alpha}}$$

Otherwise, if the utility from conflict is negative, we must be in the loss domain. Thus we have:

$$-\lambda(r_A - x)^\beta = w(p)(1-r_A)^\alpha - \lambda \cdot r_A^\beta \cdot w(1-p) - c_A \Leftrightarrow$$
$$x_A = r_A - \left(r_A^\beta \cdot w(1-p) + \frac{c_A - w(p)(1-r_A)^\alpha}{\lambda}\right)^{\frac{1}{\beta}}$$

We find the indifference point for player B in a similar manner. Combining these conditions, we arrive at the formulas for each player's indifference point:

$$x_A = \begin{cases} r_A + \left(w(p)(1-r_A)^\alpha - \lambda \cdot w(1-p) \cdot r_A^\beta - c_A\right)^{\frac{1}{\alpha}} & \text{if } w(p)(1-r_A)^\alpha - \lambda \cdot w(1-p) \cdot r_A^\beta - c_A \geq 0 \\ r_A - \left(w(1-p) r_A^\beta + \frac{c_A - w(p)(1-r_A)^\alpha}{\lambda}\right)^{\frac{1}{\beta}} & \text{otherwise} \end{cases} \tag{3}$$

$$x_B = \begin{cases} r_B - \left(w(1-p) r_B^\alpha - \lambda \cdot w(p)(1-r_B)^\beta - c_B\right)^{\frac{1}{\alpha}} & \text{if } w(1-p) r_B^\alpha - \lambda \cdot w(p)(1-r_B)^\beta - c_B \geq 0 \\ r_B + \left(w(p)(1-r_B)^\beta + \frac{c_B - w(1-p) r_B^\alpha}{\lambda}\right)^{\frac{1}{\beta}} & \text{otherwise} \end{cases} \tag{4}$$

As in section 2, the bargaining range is equal to $[x_A, x_B]$ if $x_A \leq x_B$ and is the empty set if $x_A > x_B$.

3.2 Analysis of Full Model

The analysis of the full model is more complicated than the analysis of the simple model. Many of the properties that were true above, such as proposition 1 are not true in the general case for any r_A and r_B. In addition, though changing one parameter can sometimes eliminate the bargaining range, this is not generally true. Yet, here too we can see that a collusion of the various effects of prospect theory can lead to the disappearance of the bargaining range. In particular, as the parameters of importance from prospect theory (β, δ and λ) approach their extreme values (zero for β and δ, and infinity for λ) then each player's indifference point converges into his reference point. In fact this convergence occurs when just two of the three parameters approach their extreme values. Thus if the players' reference points are conflicting, more extreme parameter values raise the likelihood of conflict.

This property can be formalized in the following proposition.

Proposition 2. *For all $0 < p, c_A, c_B < 1$ and for $i \in \{A, B\}$ the following equation holds:*

$$\lim_{q \to s} x_i = r_i$$

for all q and s that are defined as to satisfy at least two of the following three conditions:

1. $\beta \to 0$
2. $\delta \to 0$
3. $\lambda \to \infty$

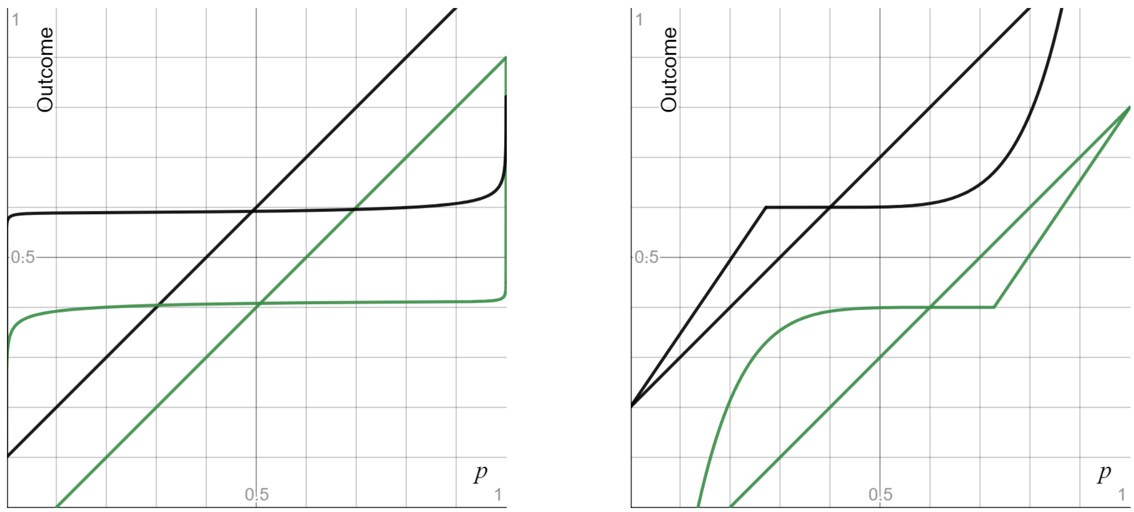
For example, using the first two conditions, we define $q = (\beta, \delta)$ and $s = (0, 0)$.

Proof: In appendix.

A similar convergence can also occur (though for a limited range of probabilities) by only manipulating β. Examples of this behavior can be found in Figure 6b, while an example of proposition 2 can be found in Figure 6a.

An immediate corollary of proposition 2 is that if the reference points are conflicting ($r_A > r_B$) then for all $0 < p, c_A, c_B < 1$ and for all combinations of the three conditions, there exist some critical values such that if the parameters are at more extreme values than these critical values then the bargaining range is empty. Formally:

Figure 6: Convergence on r_i



(a) $r_A = 0.45$, $r_B = 0.55$, $\delta = 0.2$ $\lambda = 4$ (b) $r_A = 0.4$, $r_B = 0.6$, $\beta = 0.15$

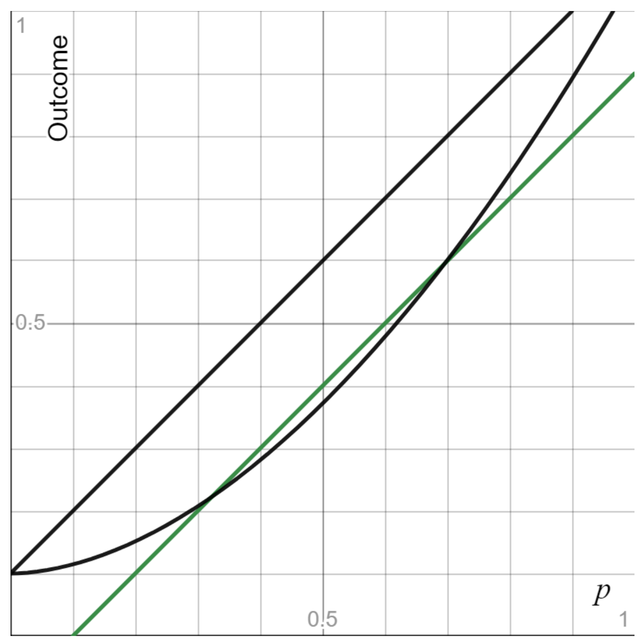
Note: For explanation of figure, see Figure 3.

Corollary 1 (Corollary to Proposition 2)**.** *If $r_A > r_B$, then for all $0 < p, c_A, c_B < 1$ there exist critical values $\tilde{\beta}_p$ and $\tilde{\delta}_p$ such that for all $\beta < \tilde{\beta}_p \cap \delta < \tilde{\delta}_p$, $BR = \emptyset$. Likewise, such critical values exist for all combinations of two parameters out of (β, δ, λ).*

Intuitively, more extreme parameter values cause the indifference points to converge on the reference points. When the reference points are past each other, this ensures that in the convergence process the indifference points must cross each other at some point, eliminating the bargaining range. The corollary demonstrates the importance of the reference points as well as the interacting effects of the parameters. Though no one parameter is necessarily sufficient to make the bargaining range disappear, as long as the reference points are conflicting then for every level of probability p, and even regardless of the costs of conflict (as long as it is less than the value of the players' preferred outcome), one can manipulate any two parameters in such a way as to eliminate the bargaining range.

However, despite the importance of conflicting reference points, conflict can occur in the model even when $x_A \leq x_B$. An example of conflict with non-conflicting reference points can be found in Figure 7. Note that since player A is always in the gain domain and $\alpha = 1$ his indifference point is identical to the indifference point of the risk-neutral agent (the linear line).

Figure 7: conflict With $r_A \leq r_B$ ($r_A = 0$, $r_B = 0.1$, $\beta = 0.55$, $\lambda = 3$)



Note: For explanation of figure, see Figure 3.

In this section we have seen that the results of the full model are not cut and dry. Unlike in section 2, it is not generally true that cost parameters can be found at which conflict is inevitable, nor is one parameter always sufficient to generate conflict. Also, despite the importance of conflicting reference points in creating conflict, it is not a necessary condition, and counter-examples can be found. What does become apparent, however, is that there is an interacting effect between the different parameters and the reference point that jointly can cause conflict. In particular, as long as the reference points are conflicting two parameters are sufficient to explain conflict.

4 Discussion and Conclusion

This paper has analyzed the importance of cognitive biases and reference points in conflict negotiations. The model demonstrates that when augmented by cognitive biases, conflicting reference points can be a major cause of conflict. In particular, I focused on the effects of diminishing sensitivity, loss aversion and probability weighting (β, λ and δ). This analysis does not discount the importance of other factors such as those discussed in Fearon (1995) (private information, credibility, etc). However, the main contribution of this paper is that conflict can be explained without resorting to the factors discussed in Fearon (1995) if players make decisions according to prospect theory and have conflicting points of reference.

The model makes some interesting predictions. For example, the model may be used to help predict the timing of conflict. Conflict can be caused by a change in reference points, a change in costs of conflict, or a change in the probability of victory. For concreteness, let's look at Figure 4a. At $p = 0.1$ we predict that a negotiated settlement could be found. If however, the chance of victory (by player A) increases to $p = 0.4$ conflict would inevitably occur. These predictions could be the focus of future research.

Lastly, this model opens new avenues of avoiding conflict that do not appear in Fearon's model. For example, sometimes working on changing the reference points of the parties may be more effective in avoiding conflict than "Fearonian" methods such as making private information public. In fact, attempts to eliminate distortions such as private information are doomed to fail in their goal to eliminate conflict if conflict is an unavoidable consequence of the players' preferences.

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A Proofs

A.1 Proof of Proposition 1

Proof. Let x_A^0 and x_B^0 be the indifference points of the two players when $c_A = c_B = 0$. Therefore:

$$x_A^0 = 1 - \left[\frac{(1-p)^\delta}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta}}} \right]^{\frac{1}{\beta}}$$

and

$$x_B^0 = \left[\frac{p^\delta}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta}}} \right]^{\frac{1}{\beta}}$$

Assume by contradiction that $x_A^0 \leq x_B^0$. Thus we have:

$$1 \leq \frac{p^{\frac{\delta}{\beta}} + (1-p)^{\frac{\delta}{\beta}}}{[p^\delta + (1-p)^\delta]^{\frac{1}{\delta\beta}}}$$

Clearly, the RHS is equal to one when $\beta = \delta = 1$. Furthermore, it can be shown that the partial derivatives of the RHS with respect to β and with respect to δ are both strictly positive. We are given that either $\beta < 1$ or $\delta < 1$, therefore the above equation is false. By contradiction, we must have $x_A^0 > x_B^0$. Furthermore, by the continuity of x_A and x_B in c_A and c_B respectively, there must exist some $c_A^*, c_B^* > 0$ such that for all $c_A < c_A^*$ and $c_B < c_B^*$ we have $x_A > x_B$. This implies that the bargaining range is the empty set for any $c_A < c_A^*$ and $c_B < c_B^*$.

□

A.2 Proof of Proposition 2

Proof. For reference here are the indifference points of the two players:

$$x_A = \begin{cases} r_A + \left(w(p)(1-r_A)^\alpha - \lambda \cdot w(1-p) \cdot r_A^\beta - c_A\right)^{\frac{1}{\alpha}} & \text{if } w(p)(1-r_A)^\alpha - \lambda \cdot w(1-p) \cdot r_A^\beta - c_A \geq 0 \\ r_A - \left(w(1-p)r_A^\beta + \frac{c_A - w(p)(1-r_A)^\alpha}{\lambda}\right)^{\frac{1}{\beta}} & \text{otherwise} \end{cases}$$

$$x_B = \begin{cases} r_B - \left(w(1-p)r_B^\alpha - \lambda \cdot w(p)(1-r_B)^\beta - c_B\right)^{\frac{1}{\alpha}} & \text{if } w(1-p)r_B^\alpha - \lambda \cdot w(p)(1-r_B)^\beta - c_B \geq 0 \\ r_B + \left(w(p)(1-r_B)^\beta + \frac{c_B - w(1-p)r_B^\alpha}{\lambda}\right)^{\frac{1}{\beta}} & \text{otherwise} \end{cases}$$

Additionally, note that $\lim_{\delta \to 0} w(p) = 0 \quad \forall 0 < p < 1$.

We will check each pair of parameter limits and prove that the limit of the indifference points is equal to the reference points. First let us check the limit when $\beta \to 0$ and $\delta \to 0$. Note that at $\beta = \delta = 0$, the utility of conflict $(w(p)(1-r_A)^\alpha - \lambda \cdot w(1-p) \cdot r_A^\beta - c_A)$ is negative. Therefore we are on the second branch of each equation. WLOG we will look at x_A.

$$\lim_{(\beta,\delta)\to(0,0)} x_A = r_A - \lim_{(\beta,\delta)\to(0,0)} \left(w(1-p)r_A^\beta + \frac{c_A - w(p)(1-r_A)^\alpha}{\lambda}\right)^{\frac{1}{\beta}} = r_A$$

Where the second equality holds because the limit of the expression inside the parentheses is c_A/λ which is a positive number less than one which tends to zero as the exponent outside the parentheses tends to infinity.

Next, we will prove the proposition when $\beta \to 0$ and $\lambda \to \infty$. Notice again that the utility of conflict is negative (and tends to minus infinity) therefore again we are on the second branch. Again we will look at x_A.

$$\lim_{(\beta,\lambda)\to(0,\infty)} x_A = r_A - \lim_{(\beta,\lambda)\to(0,\infty)} \left(w(1-p)r_A^\beta + \frac{c_A - w(p)(1-r_A)^\alpha}{\lambda}\right)^{\frac{1}{\beta}} = r_A$$

Again, the second equality holds because the expression inside the brackets tends to a positive number less than one and it is being raised to an exponent that tends to infinity.

Lastly, we will prove the proposition when $\delta \to 0$ and $\lambda \to \infty$.

$$\lim_{(\delta,\lambda)\to(0,\infty)} x_A = r_A - \lim_{(\delta,\lambda)\to(0,\infty)} \left(w(1-p)r_A^{\beta} + \frac{c_A - w(p)(1-r_A)^{\alpha}}{\lambda} \right)^{\frac{1}{\beta}} = r_A$$

Here it is easy to see that the term inside of the parentheses tends to zero.

□